Math 12 Honors: Challenging Ch3 Polynomials Problems:

1	Find the remainder when $x^{13} + 1$ is divided by $x - 1$. (AHSME 1950)
2	Find all the roots of $2y^4 - 9y^3 + 14y^2 + 6y - 63 = 0$.
3	Find all values of m which will make $x + 2$ a factor of $x^3 + 3m^2x^2 + mx + 4$. (MAΘ 1991)
4	Find the product of the nth roots of 1. (MAΘ 1991)
6	The equation $x^4 - 16x^3 + 94x^2 + px + q = 0$ has two double roots. Find $p + q$. (MAΘ 1991)

7	Let $f(x) = ax^7 + bx^3 + cx - 5$, where a, b, and c are constants. If $f(-7) = 7$, then find $f(7)$. (AHSME
8	**90.** For nonzero constants c and d, the equation $4x^3 - 12x^2 + cx + d = 0$ has two real roots which add to give 0. Find d/c. (MAΘ 1991)
9	Let r, s, and t be the roots of $x^3 - 6x^2 + 5x - 7 = 0$. Find $$\frac{1}{r^2} + \frac{1}{s^2} + \frac{1}{t^2}.$$
10	Solve the equation $(x + 1)(x + 2)(x + 3)(x + 4) = -1$. (M&IQ 3)
11	Give the remainder when $x^{203} - 1$ is divided by $x^4 - 1$. (MAΘ 1991)

1 2	Given the equation $\left(x^2 - 3x - 2\right)^2 - 3(x^2 - 3x - 2) - 2 - x = 0,$ prove that the roots of the equation $x^2 - 4x - 2 = 0$ are roots of the initial equation and find all real roots of the given equation. (Bulgaria 1993)
1 3	103. If a, b, c, d are the solutions of the equation $x^4 - mx - 3 = 0$, then find the polynomial with leading coefficient 3 whose roots are $$\frac{a+b+c}{d^2}, \quad \frac{a+b+d}{c^2}, \quad \frac{a+c+d}{b^2}, \quad \text{and} \quad \frac{b+c+d}{a^2}.$$
1 4	The roots of $f(x) = 3x^3 - 14x^2 + x + 62 = 0$ are $a, b,$ and $c.$ Find the value of $$\frac{1}{a+3} + \frac{1}{b+3} + \frac{1}{c+3}.$$
1 5	Show that if s_k is the sum of the kth powers of the roots of $a_3x^3 + a_2x^2 + a_1x + a_0,$ then $a_3s_2 + a_2s_1 + 2a_1 = 0.$
1 6	Find an equation whose roots are 3 greater than those of $x^4 - 3x^3 - 3x^2 + 4x - 6.$

1 7	Find a polynomial whose roots are twice those of $f(x) = x^4 - 3x^2 + x - 9$.
1 8	If three roots of $x^4 + Ax^2 + Bx + C = 0$ are -1, 2, and 3, then what is the value of $2C - AB$?
1 9	Find the roots of $x^4 + x^3 + 2x^2 + 17x - 21$.
2 0	Find all of the solutions to the equation $$x^4 - 10x^3 + 35x^2 - 50x + 24 = 0.$$
2 1	Let $P(x)$ be the unique polynomial of minimal degree with the following properties: • $P(x)$ has leading coefficient 1, • 1 is a root of $P(x) - 1$, • 2 is a root of $P(x - 2)$, • 3 is a root of $P(3x)$, and • 4 is a root of $4P(x)$. The roots of $P(x)$ are integers, with one exception. The root that is not an integer can be written as $\frac{m}{n}$, where m and n are relatively prime positive integers. What is $m + n$? **(A)** 41 **(B)** 43 **(C)** 45 **(D)** 47 **(E)** 49